Infectrate Simulation: Mathematical Model and Analysis

# 1. Introduction

The Infectrate Simulation models how a disease spreads through a population using an agent-based approach. It incorporates mathematical and numerical techniques to analyze the progression of infection, recovery, and death within a confined environment. The mathematical foundation allows for insightful observation of real-world disease behavior.

# 2. Project Overview

Each agent in the simulation represents an individual that can be in one of five states:  
- Susceptible (S)  
- Infected (I)  
- Recovered (R)  
- Dead (D)  
- Bacteria (B, environmental influence)  
  
Agents move and interact within a bounded space, and disease transmission occurs based on proximity and probability.

# 3. Mathematical Model

The simulation models disease spread using the following key equations and methods:

## 3.1 Differential Equations

The model is built upon rate-based transitions among compartments, similar to the SIR model. These transitions are governed by first-order differential equations:  
  
 dS/dt = -βSI/N  
 dI/dt = βSI/N - γI - μI  
 dR/dt = γI  
 dD/dt = μI  
  
Where:  
- β is the infection rate  
- γ is the recovery rate  
- μ is the death rate  
- N is the total population

## 3.2 Numerical Integration (Trapezoidal Rule)

To estimate the total number of infected individuals over time, the Trapezoidal Rule is used:  
  
 ∫ I(t) dt ≈ Σ [ (Iₖ + Iₖ₋₁) / 2 ] × Δt  
  
This helps determine the overall disease burden during the simulation duration.

## 3.3 Numerical Differentiation (Central Difference)

To estimate the rate of change of infections, the central difference method is used:  
  
 dI/dt ≈ [I(t+Δt) - I(t-Δt)] / (2Δt)  
  
This is useful in identifying the time at which the infection rate peaks.

## 3.4 Bisection Method

The simulation applies the Bisection Method to determine the time (t) when the number of infected equals the number of susceptible individuals. The method iteratively searches for the root of the function f(t) = I(t) - S(t):  
  
If f(a) \* f(b) < 0, then a root exists in [a, b]. Continue halving the interval until |f(c)| < tolerance.

# 4. Importance of Mathematical Modeling

These mathematical concepts allow for accurate, data-driven predictions of disease behavior. They inform public health decisions, simulate intervention outcomes, and provide educational insights. Combining these methods in a visual simulation brings theoretical models to life.

# 5. Conclusion

The Infectrate Simulation fuses mathematical modeling and real-time simulation to illustrate how infectious diseases spread and evolve. The integration of differential equations, numerical methods, and logical control offers both a scientific and intuitive view of epidemic dynamics.